Preparation Guide for Test 2

Test 2 will be in class on **Wednesday, 26 February**. This will use the normal class time, so you will have 50 minutes to complete the test. The test will be similar to Test 1, but focused on new content.

You can get a good idea what to expect on the test by looking at the following practice problems. The test will be shorter than the practice problems—this is meant to give you more problems to practice than you will have on the test, so don't worry if it takes longer than 50 minutes to solve them all. We strongly encourage you to try to problems on your own, before discussing the problems with others. In class on Monday (24 Feb) we will discuss any problems students have questions about.

Resources. For the test, you will be permitted to use a single paper page of notes that you prepare and bring to class. It is fine to collaborate with others to prepare your notes. The page should be no larger than a US Letter size page (8.5×11 inches), and you may write (or print) on both sides of the page. You may not use any special devices (e.g., magnifying glasses) to read your page. No other resources, other than your own brain, body, and writing instrument, are permitted during the exam.

Content. The problems on the test will cover material from Classes 1–17 (but focus more on material covered since Test 1), Problem Sets 1–4 including the provided comments (with a focus on Problem Set 3 and 4), and the MCS readings mentioned in the problem set Preparation sections. Problems on the test will emphasize things you have seen in at least two of these (Classes, Problem Sets, and MCS Book), and most problems will focus on things that are covered in all three. The purpose of the test is to see how well you understand and can apply the key concepts we have covered so far, not to test your memory of some obscure detail. If you understand the problems on the problem sets and the practice problems in this guide well enough to be able to answer similar questions, you should do well on the exam.

The main topics the test will cover are:

– Sets including how do define sets and understand sets described using set comprehensions ("set builder notation").

– Operations on sets — be able to use definitions of set operations, including both ones we have covered as well as being able to devise and interpret similar types of definitions.

– Logical quantifiers — understand propositions using $\forall$ and $\exists$, and be able to negate and construct proofs involving these quantifiers.

– Binary relations — understand what a binary relation is and how to define one, and be able to use the binary relation properties (function, total, injective, surjective, and bijection).

– Cardinality — understand how we define the cardinality of finite sets, and be able to prove properties about set cardinalities.

– Set theory and natural numbers — understand how we can define the natural numbers from simple set operations, and how to reason about and definte operations on natural numbers defined this way.

For most students, we believe the best way to prepare for the exam are the same as those we described on the Preparation Guide for Test 1.

Practice Problems

Problem 1 *Basic Set Questions*

Write the value of each of the following expressions.

(a) $\{0, 1, 2\} \cup \{2\} =$

(b) $\{0, 1, 2\} \cap \{2\} =$

(c) $\{(2x, x) \mid x \in \mathbb{N}, x < 3\} =$

(d) $\mid pow(\{92, 2\}) \mid=$

(e) $\{1, 3\} \times pow(\{0, 1\}) =$

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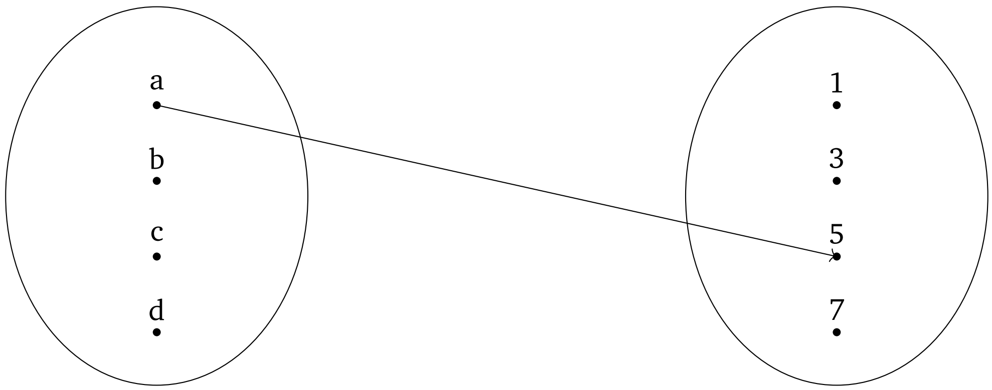
Problem 2 *Drawing Binary Relations*

For the diagram below, assume $\{a, b, c, d\}$ is the entire domain and $\{1, 3, 5, 7\}$ is the entire codomain.



(a) Add arrows to the diagram below, to make the relation surjective, but not injective.

(b) Add or remove arrows from the diagram you created in the previous subproblem to make the relation injective.

(c) Is the relation a total function? If not, add or remove as few arrows as possible to make the relation a total function.

(d) Add an arrow to the diagram that makes it so the diagram is not a binary relation.

Problem 3 *Binary Relations*

Explain why the binary relation $R = (\mathbb{R}, \mathbb{R}, G = \{(x, 1/x) \mid x \in \mathbb{R}\}$ is not *surjective*.

Problem 4 *Set Equality Proof*

For any finite sets A, B, prove:

$$A = (A - B) \cup (A \cap B)$$

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Problem 5 *Binary Relations Proof*

Prove that for any three finite sets, A, B, and C, if there exists a surjective total function between A and B, and a surjective total function between B and C, there must exist a surjective total function between A and C. (Hint: think about what the existence of a surjective total function means about the set cardinalities.)

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Problem 6 *Quantifiers and Relations*

Create an alternate but equivalent definition of *surjective* for a binary relation, $R = (A, B, G \subseteq A \times B)$, without talking about arrows.

Some operators that may be useful include $\forall$, $\exists$, $\neg$, $\in$.

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Problem 7 *Natural Numbers*

Assuming Cartesian product of sets is defined, for any sets A and B

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Consider the following alternative way of defining the natural numbers:

“0”$= \{\emptyset\}$
successor$(n) := n \times n$

Can we define a set that corresponds to the natural numbers using this representation of “0” and the *successor* function as defined above instead of the one we did in class? Explain either how numbers would be represented, or any problems that would occur using this definition (and even better if you can explain how to fix them).