

Problem Set 2: Methods of Proof

Collaborators and Resources: TODO: replace this with your collaborators and resources (if you did not have any, replace this with *None*)

Submit your responses as a single PDF file to gradescope before **8:29pm** on **Thursday, 30 January**.

You are strongly encouraged to start early and take advantage of the scheduled office hours for this course.

Collaboration and Resources Policy - Read Carefully

Remember to follow the course pledge you read and signed at the beginning of the semester.

For this assignment, you may discuss the problems and work on solutions with anyone you want (including other students in this class), but you must write your own solutions and understand and be able to explain all work you submit on your own.

To confirm your own understanding, after discussing the problems with others, you should attempt to write your solutions on your own without consulting any notes from group work sessions. If you get stuck, you may visit notes from the group work sessions, but should make sure you understand things well enough to produce it on your own. You may also use any external resources you want, with the exception of solutions and comments from last year's offering of this course.

Since the staff and students benefit from being able to both reuse problems from previous years, and from being able to provide detailed solutions to students, it is important that students do not abuse these materials even if it is easy to find them. Using solutions from last year's course would be detrimental to your learning in this course, and is considered an honor violation.

If you use resources other than the class materials, lectures, and course staff, you should document this and mention it clearly on your submission. For everyone other than the course staff you work with, you should credit them clearly on your assignment. If you use any AI tools like ChatGPT or Claude (which we do encourage, so long as you are using them to learn!), you should explain how they used them and include a URL that links to a transcript of your interactions.

Preparation

This problem set focuses on Chapter 1 of the *MCS book*, and Classes 1–6 (focusing on material after PS1).

Directions

(Identical to PS1, other than the template URL)

1. Follow the steps as in Problem Set 0 to create your own copy of the template in <https://www.overleaf.com/read/wckvjmxvfsc#a6fe27>.
2. Solve all the problems and put your responses in the clearly marked answer boxes. For full credit, your answers should be correct, clear, well-written, and convincing.
3. Before submitting, make sure to list your collaborators and resources by replacing the TODO in `\collaborators{TODO: replace ...}` with your collaborators and resources. Check the policy in the pink box on the front page to make sure you understand what you need to document here.
4. Replace the second line in `ps2.tex`, `\usepackage{dmt}` with `\usepackage[response]{dmt}` so the directions do not appear in your final PDF.
5. Download your complete `ps2.pdf` file, and submit it using gradescope.

Problem 1 *An Odd Proof*

As mentioned in Class 5, prove the *Even-Odd Lemma* we needed in our proof of the Even Square Theorem:

Even-Odd Conjecture: For any natural number n , if n is not even, n is odd.

Hint: there is an easy way to prove this in much less space than the answer box. As with any problem in this class, unless we explicitly state otherwise, you can always use any theorem we have proven in class in your proofs.

Problem 2 *Critiquing Claude*

Consider the proof below, generated by Claude-3.5 Sonnet (You can follow this link to see the transcript and continue interactions, but this is not necessary and requires a Poe account: <https://poe.com/s/nSYteW5bchjFDXn2E1Qg>.)

Human User:

Prove: If a triangle equilateral, it is not isosceles.

 **AI:**

Let me help you prove why this statement is actually false. I'll use a proof by contradiction.

Given statement: "If a triangle is equilateral, it is not isosceles"

Proof:

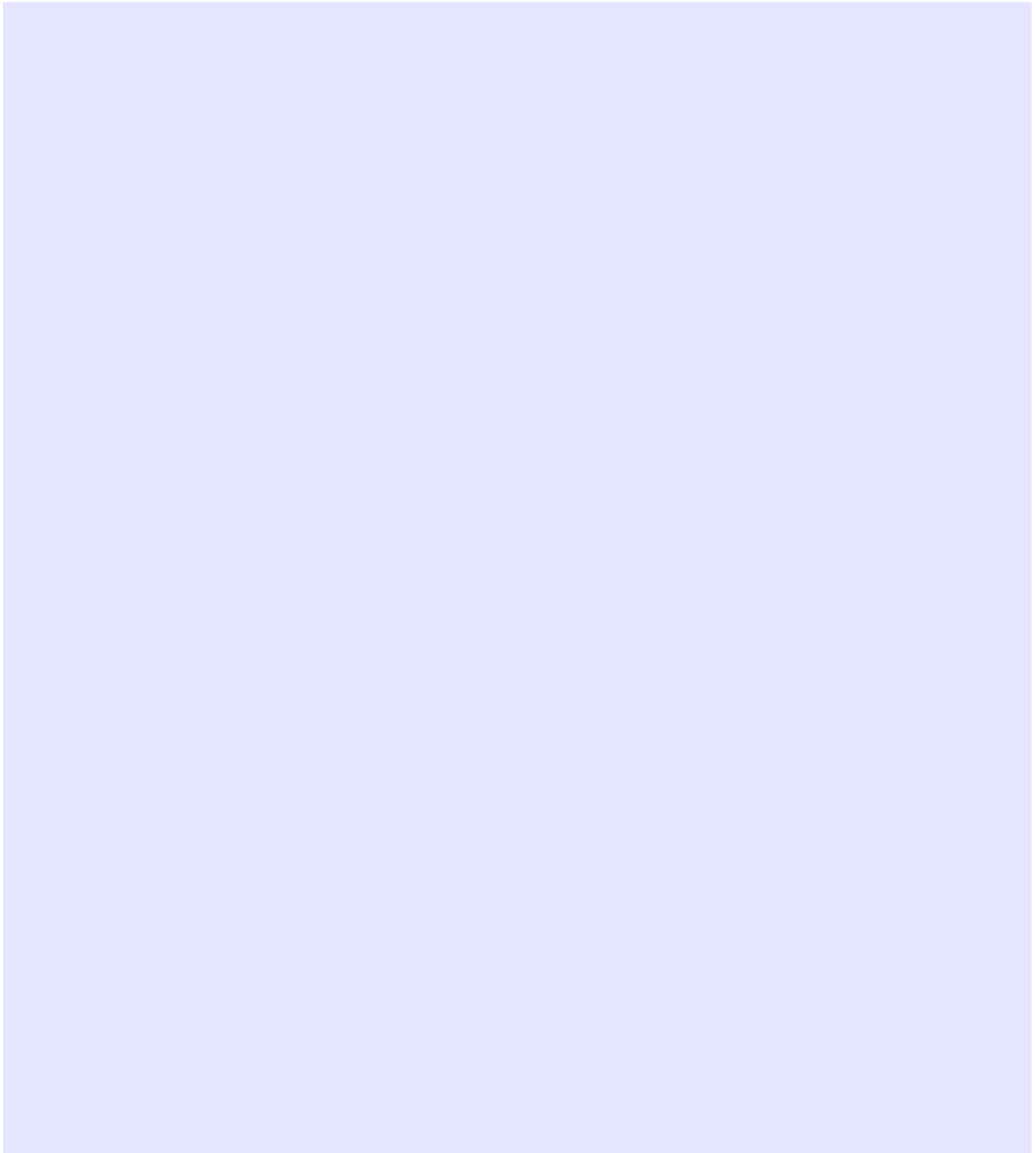
1. Let's assume the given statement is true
2. This means that if a triangle is equilateral, it cannot be isosceles
3. By definition:
 - An equilateral triangle has all 3 sides equal
 - An isosceles triangle has at least 2 sides equal
4. Consider an equilateral triangle ABC where:
 - $AB = BC = AC$ (all sides are equal)
5. Since $AB = BC$, the triangle has at least two equal sides
6. By definition of isosceles triangle, ABC is isosceles
7. This contradicts our initial assumption that an equilateral triangle cannot be isosceles

Therefore, the original statement is false.

In fact, every equilateral triangle is also isosceles, since an equilateral triangle is a special case of an isosceles triangle where all three sides are equal instead of just two.

Would you like me to explain any part of this proof in more detail?

Critique Claude's proof: (a good answer will point out at least one serious problem)



Problem 3 *Rationality Proof*

Prove that $\sqrt{16}$ is rational.



Problem 4 *Irrationality Proof*

Prove that $\sqrt{5}$ is irrational.



Problem 5 *Rooting out Squares*

Prove that for any non-negative real numbers, x and y , if $xy = n$ then the minimum of x and y is not greater than \sqrt{n} . (Hint: prove by contradiction.)

End of Problem Set 2!

Remember to follow the instructions to prepare and submit your PDF (including using *[response]* to remove the directions and completing *\collaborators* with information on your collaborators and the resources you used.